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AN INVESTIGATION INTO THERMAL BOUNDARY LAYER GROWTH IN THE ENTRANCE REGION OF AN ANNULUS

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Abstract—The paper describes an experimental investigation into the effect of an unheated length and the annulus ratio on the variations in heat transfer coefficient in the early entrance region of an annulus. The shape of the unheated section is shown to have a great influence in the first 1–2 equivalent diameters from the leading edge but thereafter, the results approach the values corresponding to the fully developed conditions rapidly.

The effect of high heater to fluid temperature ratios is also investigated and it is shown that the results are not adequately covered by the correlations commonly in use in the literature.

NOMENCLATURE

D_o, annulus outer diameter;
D_i, annulus inner diameter;
d_e, equivalent diameter ($= D_o - D_i$);
Lu, length of unheated section;
Nu, Nusselt no.;
Pr, Prandtl no.;
Re, Reynolds no.;
T, temperature;
x, distance from the leading edge.

Subscripts
b, bulk of fluid;
f, mean film condition;
s, surface of heater.

INTRODUCTION

THE PROBLEM of thermal boundary layer growth has been studied in the literature in the entrance region of circular tubes and also on flat plates but because of its application to nuclear reactor channels there is some interest in these problems when the duct is annular.

In addition to this, there have been a number of reports showing the ratio of the temperatures of the heater surface to the bulk of the fluid to have a serious influence on the heat transfer problem but there is considerable discrepancy in the findings of the different authors.

The present investigation was originally for a study of the boundary layer growth in the very early entrance region of an annular channel but was later extended to include also the effect of high surface to fluid temperature ratios.

EXPERIMENTAL APPARATUS

General arrangement

The general arrangement of the apparatus is shown diagramatically in Fig. 1a. As described elsewhere [1], air is supplied to the test section by a 45 h.p. Keith Blackman blower C, through the orifice plate, OP, and is finally discharged to the outside atmosphere.

The Reynolds number range covered by the above arrangement was $2{\cdot}0 \times 10^4$–$3{\cdot}7 \times 10^5$, based on the mean hydraulic diameter. For experiments at lower ranges of Reynolds number, air was sucked through the test section by means of a centrifugal blower downstream of it (see Fig. 1).

The main heater section consisted of an annulus in which the inner surface was electri-

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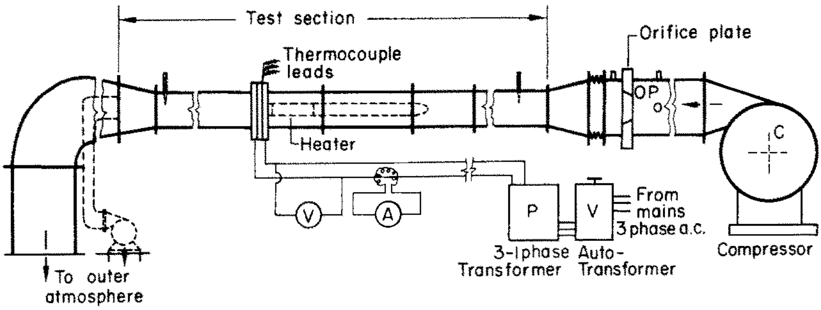

FIG. 1a. General arrangement of the test rig.

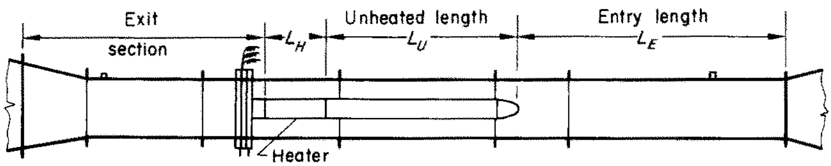

FIG. 1b. Arrangement of the test section.

cally heated. As shown in Fig. 1b, the upstream section consisted of a simple entry length and included provision for placing an unheated section or profile in front of the main heater. The exit section consisted of a mixing length in which the heated stream was made to mix, and a 5° exit cone.

The heater was of 1·57 in. (39·89 mm) o. d. and to investigate the influence of the annulus ratio on the results, two separate sets of outer tubes, having diameters of 4 in. (101·6 mm) and 2·75 in. (69·85 mm) were employed.

To minimize the heat loss from the heater to outer annulus surface by radiation, the surfaces of the outer annulus in the sections surrounding the heater, as well as the section immediately behind it, were chromium plated and the entire test section was wrapped by a 1 in. thick insulation of corrugated asbestos paper.

Heater section

A cross sectional view of the main heater is shown in Fig. 2a, while a pictorial representation of the element assembly is shown in Fig. 2b.

The heater consisted of a very thin walled stainless steel tube H, through which a current of up to 600 A was conducted longitudinally. The tube was fabricated by rolling a 0·0762 mm (0·003 in.) thick sheet into shape and spot welding along the longitudinal joint, the ends being welded to cylindrical support T and ferrule D. Power was supplied to the element through spiders S_2, S_1 and a central rod R, which also combined to provide rigid support for the assembly. The materials of the heater and inner rod were cycled to 700°C before fabrication into actual components. Before rolling thermocouples were spot welded to the inner surface of the heater, special care being taken to ensure that the welding produced no perceptible distortion of the outer surface.

Relative movement between the central electrode R and the heater surface, caused by differential thermal expansion was accommodated by disc springs DS. Any electrical heating of the springs was avoided by by-passing them electrically, using heavy copper braids B.

The ferrule D was thinned to 0·8 mm ($\frac{1}{32}$ in.)

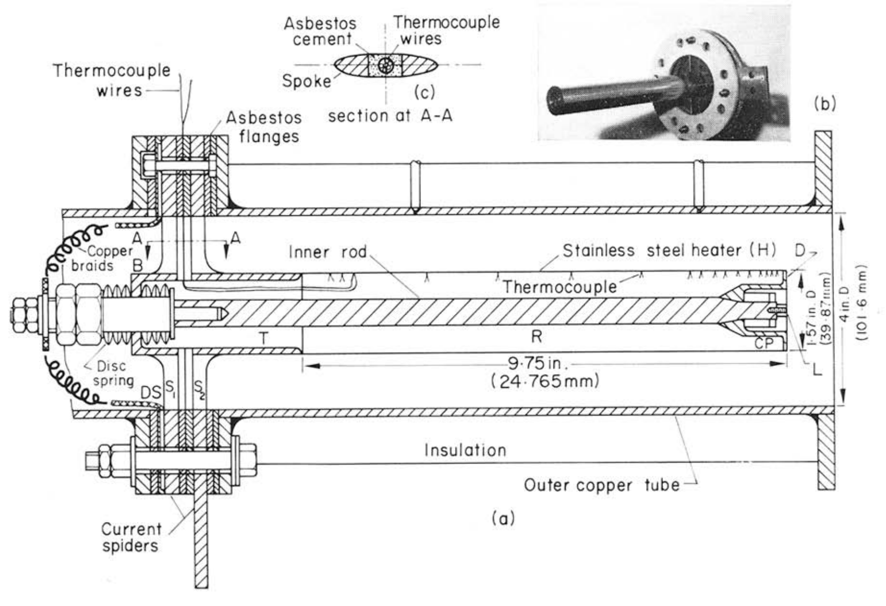

FIG. 2. Heater assembly.

near its pheripheral weld with the tube, thus reducing the heat conducted away from the tube end. Attachment to the central rod was by a Morse taper, and nut.

The design allowed for three different geometrical configurations preceding the heater section: a flat front, an elliptical nose-piece with major and minor axes of 127 and 39·89 mm respectively and an unheated cylindrical length of the same diameter as the heater.

Following the results of the experiments at low surface temperatures, the effect of high surface temperatures were investigated using the unheated length and the elliptical nose-piece only.

Instrumentation

The thermocouples used in the high temperature parts of the apparatus, namely the heater tube, H, central rod R and cap CP, were made from Platinum and Platinum + 13% Rhodium wires, while those used for the outer copper tube and the insulation were of Chromel and Alumel wires.

Interference with the thermal pattern and distortion of the surface were both minimised at the junction by using very thin wires (0·0762 mm dia.) for the first 2·5 cm of the thermocouple leads, the diameter thereafter being increased to 0·127 mm. These wires were insulated by fibre glass sleeves which proved to be satisfactory at temperatures well above the working maximum of 800°C. The air inlet and outlet temperatures were measured by means of mercury in glass thermometers.

On completion of the investigation several small pieces of the heater surface each carrying a thermocouple were cut out for calibration at the U.K.A.E.A. Laboratories and excellent agreement was obtained with published results.

The powers dissipated in, for example, the inner rod or the spiders were calculated from direct readings using a digital voltmeter. For this purpose the thermocouple wires were used as connecting points to the different parts of element and proved satisfactory in practice.

EXPERIMENTAL RESULTS

Introduction

The experimental results to be presented can be divided into two broad categories namely, those associated with the effect of geometrical configurations and of the influence of the heater surface to fluid temperature ratios.

Because of some degree of interdependence between the geometrical and temperature effects, the former parameters were investigated at comparatively small temperature differences between the heater and the fluid. While the investigation of the temperature effect consisted of the repetition of the earlier experiments, at a series of different temperature ratios.

In each of the family of measurements, the temperature of both the working fluid and the heater surface were kept approximately constant and variations of Nusselt number along the length of the heater determined for a series of constant Reynolds numbers.

The rate of heat transfer between the element and the fluid was calculated at each position along its length from a knowledge of the local rate of power generation after correction. These were due to heat exchange between the heater surface and inner rod by radiation and natural convection, the outer annulus by radiation, and conduction along its length.

The conduction loss was calculated by graphical differentiation but variations in the resistivity of the heater material with temperature, the heat exchange with the inner rod as well as the emissivity of the surfaces of the outer annulus, the heater element and the inner rod had to be determined experimentally before the start of the main programme of work.

A series of observations were also carried out to confirm that the exit geometry had no influence on the results and that the entry section was of sufficient length not to affect the measurements.

Effect of the unheated section

A comparison between the variations in Nusselt number along the heater length for the three geometrical configurations of the unheated

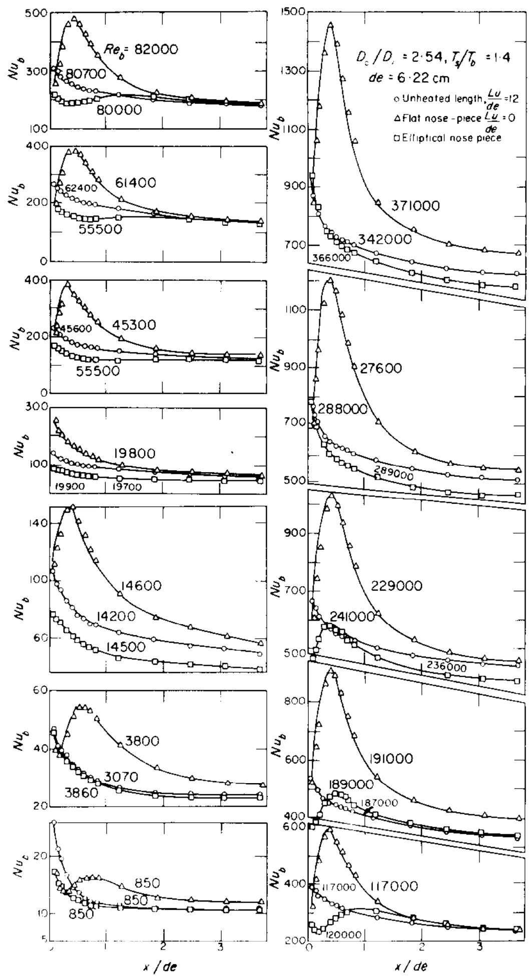

FIG. 3. Effect of the configuration of unheated section.

section, annulus ratio of 2·54, and a series of constant Reynolds numbers is given in Fig. 3. It will be shown, in connection with the results of the high temperature trials that best correlation is obtained when the properties of the fluid are taken at the local mean bulk condition. Adopting this the above measurements at an x/d_e of 3·7 are shown as plots of $Nu/Pr^{0{\cdot}4}$ against Reynolds number in Fig. 4. It will be seen that over the turbulent range the experimental observations are close to the dotted line representing the case of fully developed conditions even at a distance of 3·7 equivalent diameters from the leading edge.

With reference to Fig. 3 and taking the case of an unheated length of 12 equivalent diameters preceding the heater, the local value of the Nusselt number is high at the leading edge of the heater but diminishes with distance from it, approaching asymptotically the fully developed condition. These results are in excellent agreement with the findings of Quarmby [3].

The effect of the elliptical nose-piece is to lower the heat transfer rate over the transition stage and extend it over a longer section of the heater. The point of boundary layer transition first occurs over the heater section at a Reynolds number of $2{\cdot}0 \times 10^4$, the point of separation moving upstream with the increase of Reynolds number and finally passing upstream of the element at a Reynolds number of approximately $1{\cdot}5 \times 10^5$. Beyond this stage, Nusselt number diminishes gradually with the distance from the leading edge. These may be compared with the results of studies having bell mouthed entries in tubes. The latter investigations are by Boelter *et al.* [4], Mills [5] and Beck *et al.* [6] and despite the differences between the observations in the transition region, the agreement in the developed part is satisfactory.

In the case of the flat nose-piece a rapid transition effect occurs over the first equivalent diameter downstream of the leading edge which corresponds with wake formation caused by the abrupt change. Beyond this zone, Nusselt number decreases with distance from the leading edge and as shown in Fig. 4 it approaches the fully developed value at x/d_e of 3·7 even for this geometry. These results are similar in nature and show the same general trends as the variations in the heat transfer rate in circular tubes caused by a sudden change of section, studied experimentally by Ede *et al.* [7, 8] Jacob [9], Krall and Sparrow [10], Filetti and Kays [11], Boelter *et al.* [4] and Mills [5].

Effect of surface to bulk fluid temperature ratio

In view of the large variations in temperature across the duct section and its effect on the properties of the fluid used in Nusselt and

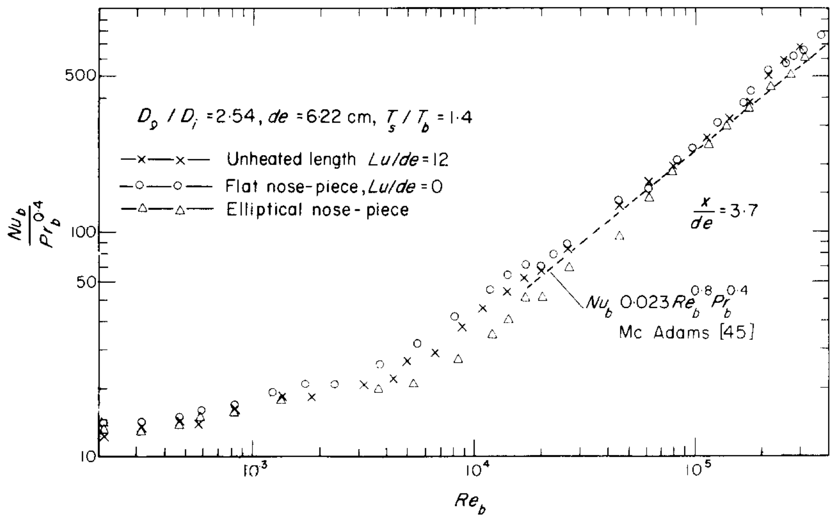

FIG. 4. Effect of the configuration of unheated section.

Reynolds members associated with the flow, it was necessary to decide on the location at which they should be based. The natural possibilities for this were the bulk of the fluid, the film mean temperature and the heater surface temperature. To assess the importance of the choice, the values of Nusselt number at $x/d = 3{\cdot}7$ are plotted against the Reynolds number, for the case of the elliptical nose-piece, preceding the heater in Figs. 5a, b and c, when fluid properties were taken at heater surface, mean film and bulk temperatures respectively. An identical set of curves were also obtained for the case of unheated length preceding the element.

It will be seen from Figs. 5a and b that the values of Nusselt number increase with increase of temperature ratio when the fluid properties are taken at surface or mean film temperatures with less variation being found in the latter case. In contrast with this as shown in Fig. 5c, Nusselt number decreases slightly with the increase of temperature ratio, when fluid properties are taken at its bulk temperature. For this reason in the results and comparisons presented, the values of the dimensionless parameters are based on properties taken at the bulk fluid temperatures.

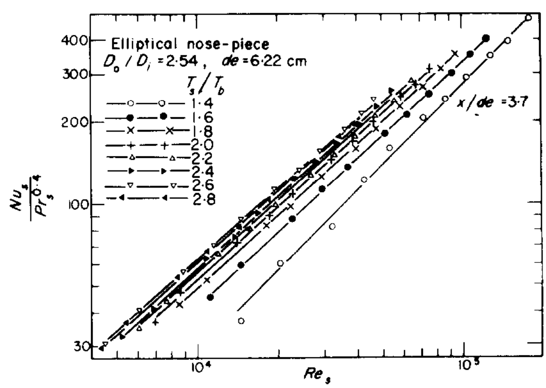

FIG. 5a. Correlation when the fluid properties are taken at surface temperature.

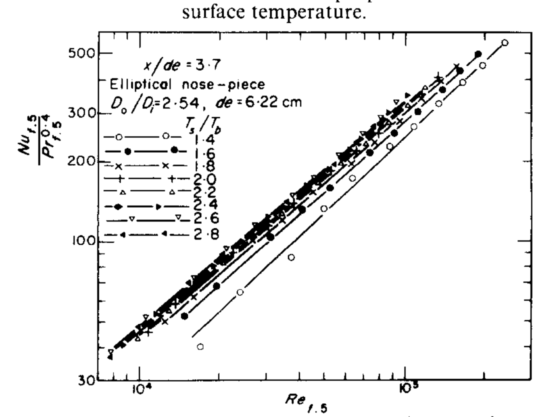

FIG. 5b. Correlation when the fluid properties are taken at mean film temperature.

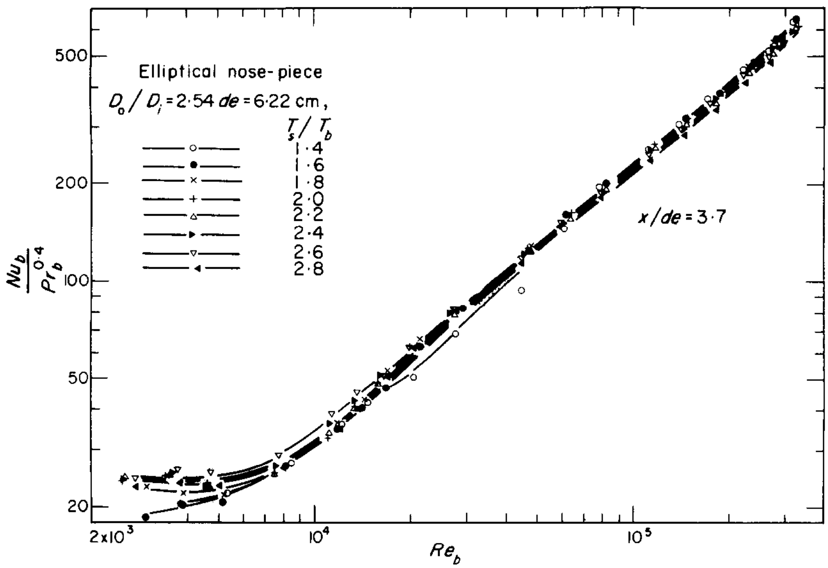

FIG. 5c. Correlation when properties are taken at bulk of fluid temperature.

Adopting this procedure, variations of Nusselt number with x/d_e as contours of constant temperature ratio, a series of constant Reynolds numbers and for each combination of unheated section and annulus ratio, are shown in Figs. 6, 7, 8a and 8b. In order to facilitate comparison and avoid small differences in Nusselt number when the Reynolds numbers are not exactly matched, the results given in the above figures are very slightly normalised.

Considering the results for the annulus ratio of 2·54 and with reference to Figs. 6 and 7 it can be seen that excepting in the very short transition region over the range x/d_e = 0·0–0·7 when the elliptical nose-piece is placed in front of the heater the effect of Reynolds number on heat transfer coefficients is identical with the low temperature trials.

The short transition region experienced with the elliptical nose-piece is affected by the temperature ratio as will be seen from Fig. 7. The position of the peak Nusselt number shifts slightly in the direction of flow, as the temperature ratio is increased.

With reference to Figs. 6 and 7, Nusselt number decreases slightly as the temperature ratio is increased. The trends are similar in the results for the annular ratio of 1·75 shown in Fig. 8 but there is a greater variation in the Nusselt number with temperature ratio for this geometry.

To separate out geometrical and temperature effects values of $Nu/Pr^{0.4}$ at x/d_e of 3·7 were plotted against Reynolds number and apart from the variations due to the temperature ratio, the results for both annular ratios fell on the same curve. Thus it was confirmed that the influence of the annulus ratio was adequately covered by the use of an equivalent diameter at small temperature differences.

To investigate the temperature effect, the values of $Nu/(Re^{0.8}Pr^{0.4})$ divided by the same parameter at T_s/T_b of 1·5 are plotted against T_s/T_b for constant values of Reynolds number and x/d_e. The results for the annulus ratio of 2·54 and Lu/d_e of 12 and the elliptical nose-pieces are plotted as Figs. 9a and b respectively. The corresponding results for the annulus ratio of 1·75 are given as Figs. 9c and d.

The series of points falling below the average curve in Fig. 9b for the case of the elliptical nose-piece in front of the heater correspond to the variations in $0 < x/d_e < 1$ range associated with this geometry. For this reason the readings corresponding with this range are omitted from the correlations in Fig. 9d. Apart from this transition effect there is a definite negative slope values of m for the case of elliptical nose-piece and unheated length for the annulus ratio of 2·54 are $-0{\cdot}104$ and $-0{\cdot}0825$ and for the annulus ratio of 1·75 they are $-0{\cdot}1824$ and $-0{\cdot}1333$ respectively. These may be compared with values of $m = -0{\cdot}29$ obtained by Dalle-

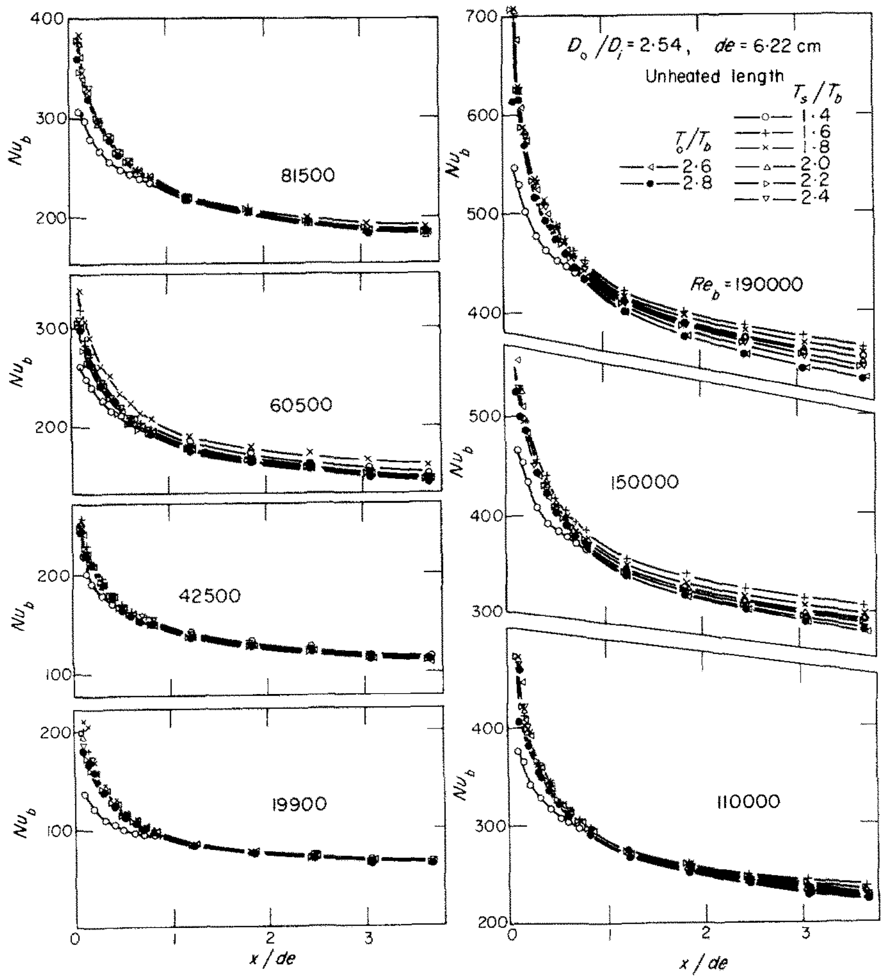

FIG. 6. Effect of temperature ratio, T_s/T_b.

of line of best fit to the entire data, but the slope varies with the configuration. Adopting an expression of the form:

$$Nu_b = 0{\cdot}023\, Re_b^{0{\cdot}8} Pr^{0{\cdot}4} \left(\frac{T_s}{T_b}\right)^m$$

Donne [12] for an annulus ratio of 2·18 at $x/d_e = 30$ and $m = -0{\cdot}15$ at $x/d_e = 5$ obtained by Petukhov [13] for a circular tube, and $m = -0{\cdot}40$ obtained by Barnes and Jackson [14] for fully developed conditions in tubes.

It is therefore, concluded that the results of heat transfer with large temperature differences can not be adequately fitted into a simple expression covering all cases. Furthermore, the apparent interdependence between geometrical and temperature effects casts some doubt on the use of modelling as a tool in design calculations. For this reason further work on the subject is necessary.

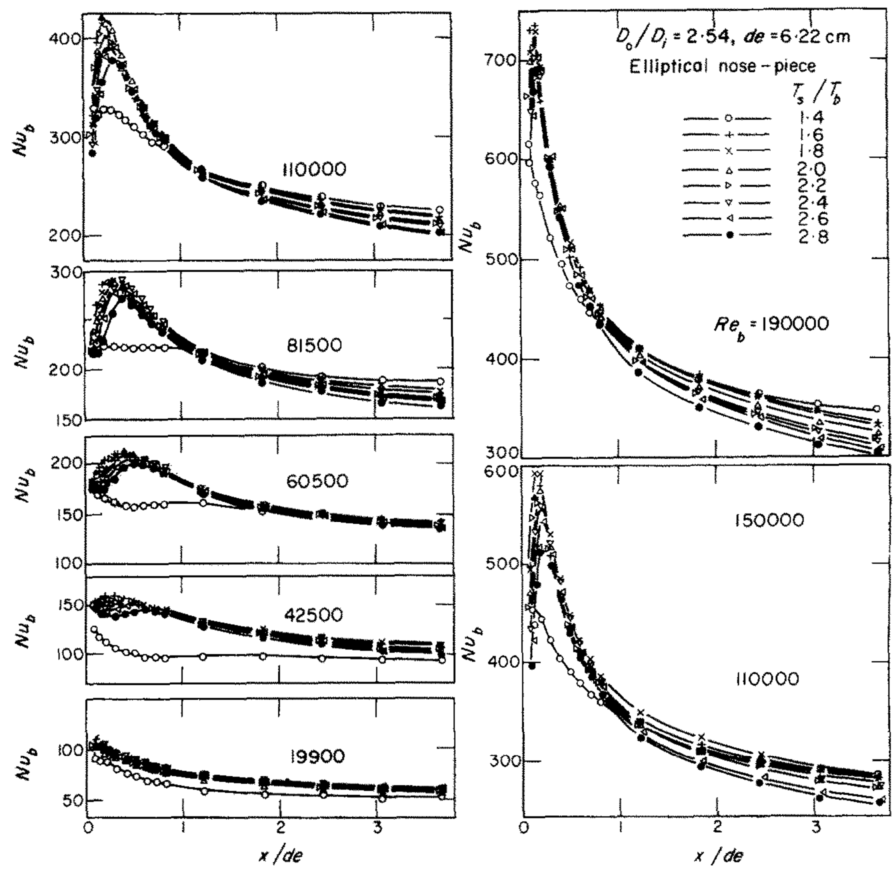

FIG. 7. Effect of temperature ratio.

SUMMARY OF CONCLUSIONS

The local heat transfer coefficients along short heater lengths are very dependent upon the geometry of the section preceding the heater. When an unheated section of sufficient length is placed upstream of the heater, the heat transfer coefficient will be high at the leading edge of the element but will drop uniformly with distance from it, assuming its fully developed value in a distance of about 4–8 equivalent diameters.

When no unheated surface precedes the heater, there will be a short transition region experienced in the first equivalent diameter from the leading edge. The heat transfer rate will reach a maximum initially but thereafter drops to its fully developed value. It will generally be above those of the previous case.

A short smooth unheated section such as an

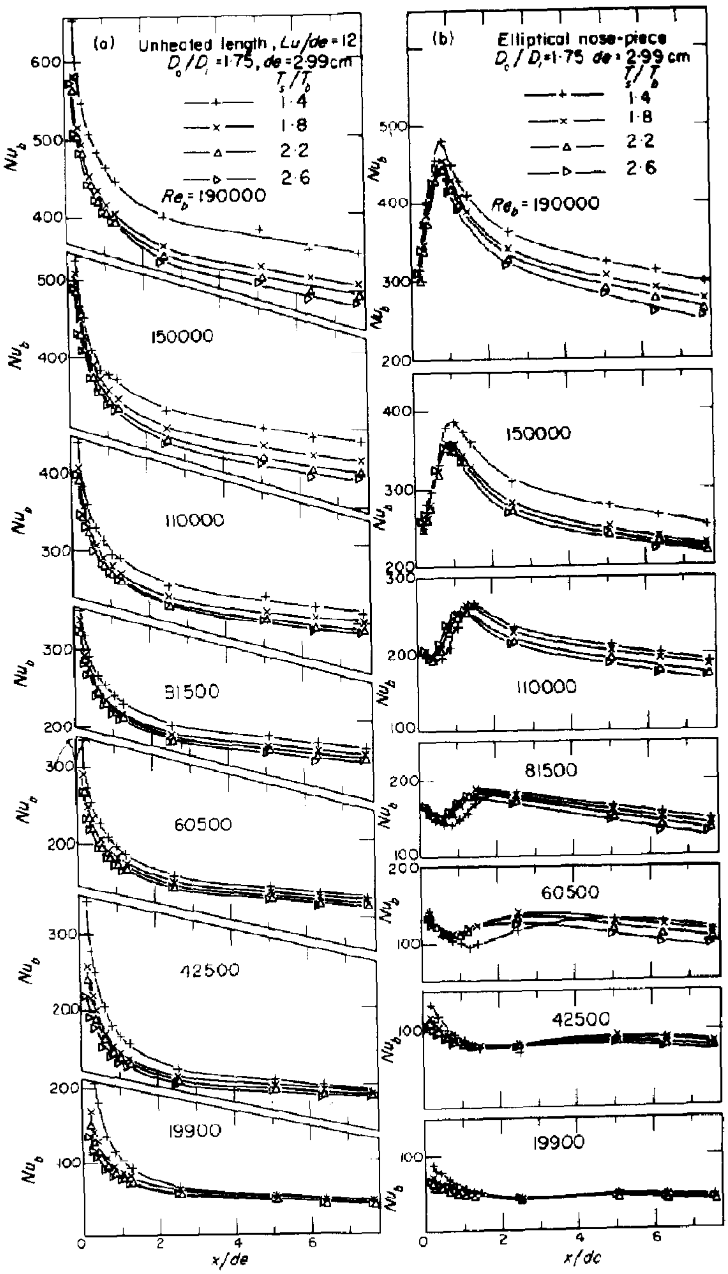

FIG. 8. Effect of temperature ratio for annulus ratio $D_0/D_1 = 1{\cdot}75$.

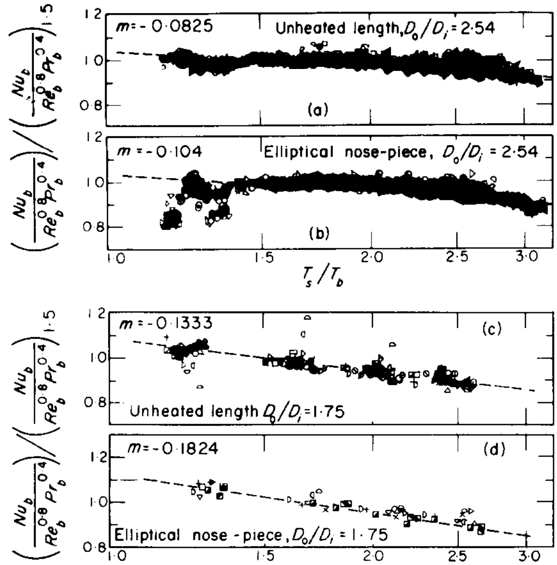

FIG. 9. Effect of temperature ratio.

elliptical nose-piece reduces the high fluxes experienced at short distances from the heater leading edge and generally causes a more uniform heat transfer rate over the heater surface.

The ratio of the temperatures of the heater surface to the fluid affects the heat transfer rate but the result depends upon the position at which the fluid properties are taken in the correlations. If the properties of the fluid are taken at the heater surface temperature an increase in Nusselt number will be observed as the temperature ratio is increased. The reverse will be true if the properties are taken at bulk fluid condition.

The influence of the annulus ratio is adequately covered by the use of equivalent diameters where the temperature difference between the heater and fluid is small. But the degree to which the temperature ratio of the heater surface to fluid bulk affects the results appears to depend upon the annulus ratio and physical dimensions and further studies of this problem are necessary.

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ETUDE DE LA CROISSANCE D'UNE COUCHE LIMITE THERMIQUE DANS LA REGION D'ENTREE D'UN ESPACE ANNULAIRE

Résumé—L'article décrit une recherche expérimentale sur l'effet d'une longueur non chauffée et du rapport des rayons, sur les variations du coefficient de transfert thermique dans la région d'entrée d'un espace annulaire. On montre que la forme de la section non chauffée a une grande influence sur les deux premiers diamètres équivalents depuis le bord d'attaque, mais ensuite les résultats approchent rapidement les valeurs correspondant aux conditions entièrement développées.

On a aussi étudié l'effet d'un chauffage interne sur les rapports de température du fluide et on montre que les résultats ne sont pas décrits de façon adéquate par les lois communément utilisées.

EINE UNTERSUCHUNG DES WACHSTUMS DER TEMPERATURGRENZSCHICHT IM EINTRITTSBEREICH EINES RINGRAUMES

Zusammenfassung—Die Arbeit beschreibt eine experimentelle Untersuchung des Einflusses einer unbeheizten Strecke und des Ringraumverhältnisses auf die Änderung der Wärmeübergangskoeffizienten im Eintrittsbereich eines Ringraumes. Die Form des unbeheizten Querschnitts hat einen grossen Einfluss auf die ersten 1–2 äquivalenten Durchmesser (gemessen vom Eintritt), aber danach nähern sich die Ergebnisse rasch den Werten der voll ausgebildeten Grenzschicht.

Der Einfluss starker Heizung auf die Fluidtemperaturverhältnisse wurde auch untersucht und es wird gezeigt, dass die Ergebnisse nicht hinreichend durch die im allgemeinen in der Literatur gebräuchlichen Wechselbeziehungen gedeckt werden.

ИССЛЕДОВАНИЕ ТЕПЛОВОГО ПОГРАНИЧНОГО СЛОЯ НА ВХОДНОМ УЧАСТКЕ КОЛЬЦЕВОГО КАНАЛА

Аннотация—Выполнено экспериментальное исследование влияния длины предвключенного участка и величины кольцевого зазора на коэффициент теплообмена. Показано, что влияние формы предвключенного участка существенно на расстоянии 1–2 калибров от передней кромки, однако дальше результаты приближаются к значениям, соответствующим полностью развитому течению. Проведено также исследование влияния нагрева сверху на температурные коэффициенты жидкости. Показано, что общепринятые в литературе соотношения непригодны для удовлетворительного описания результатов.